CPT-94/P.3116
IJS-TP-95/5
1995

Saturation of counterterms by resonances in $K \to \pi e^+ e^-$ decays

S. Fajfer

Centre de Physique Theorique - Section II - CNRS - Luminy, case 907, F-13288 - MARSEILLE CEDEX 09 - France
and
"J. Stefan" Institut, University of Ljubljana, 61111 Ljubljana, Slovenia

ABSTRACT

The decays $K^+ \to \pi^+ e^+ e^-$, $K_S \to \pi^0 e^+ e^-$ and $K_L \to \pi^0 e^+ e^-$ are reinvestigated within the framework of chiral perturbation theory. The counterterms induced by strong, electromagnetic and weak interactions are determined assuming the resonance exchange. The weak deformation model, the factorization model and the large N_c limit are used to create a weak Lagrangian. It is found that the results of the first two approaches depend on the H_1 coupling, defined in the effective chiral Lagrangian of the $O(p^4)$ order. The set of parameters used in the extended Nambu and Jona-Lasinio model can accommodete $K^+ \to \pi^+ e^+ e^-$ decay rate within the factorization approach. The CP violating $K_L \to \pi^0 e^+ e^-$ decay rate is discussed.

1 Introduction

The decays $K \to \pi e^+ e^-$ inspire many theoretical and experimental studies due to possibility to observe CP violation [1, 2, 3, 4, 5]. There are following possible decays: $K^+ \to \pi^+ e^+ e^-$, $K_S \to \pi^0 e^+ e^-$ and $K_L \to \pi^0 e^+ e^-$. The CP conserving $K^+ \to \pi^+ e^+ e^-$ and $K_S \to \pi^0 e^+ e^-$ are dominated by virtual photon exchange. The amplitudes of order $O(p^2)$ vanish at the tree level [2] and therefore the leading amplitudes for these transitions are of $O(p^4)$ order in the chiral perturbation theory (CHPT). At this order there are both one-loop contributions and tree level contributions. The one-loop contribution was determined by G. Ecker, A. Pich and E. de Rafael [1, 2, 3, 4], but the tree level contribution (or better the counterterm contribution) motivates many studies [5, 6]. The decay $K_L \to \pi^0 e^+ e^-$ proceeding via virtual γ^*, is forbidden in the limit of CP conservation. The CP conserving process proceeds through two photon exchanges. A CP violating term is proportional to the known ϵ parameter [3]. In addition, there is a direct $\Delta S = 1$ CP violating effect [5, 7]. The amplitudes for processes $K^+ \to \pi^+ \gamma\gamma$ and $K_S \to \pi^0 \gamma\gamma$ involve the same counterterm couplings as processes with one virtual photon exchange. The effective theory contains a large number of unknown parameters. The phenomenological parameters appearing in the strong sector can all be determined, while many of the weak couplings cannot be fixed by experiment.

The authors of ref. [8] have considered the resonance contribution to the coupling constants of the $O(p^4)$ effective chiral Lagrangian. They find clear evidence for the importance of vector meson contributions, which account

for the bulk of the low-energy coupling constants. The same idea has been questioned in the weak interactions sector [9, 10, 11, 12, 13]. The authors in [9, 12, 13] have studied the most general case of the $SU(3) \times SU(3)$ invariant chiral Lagrangian using symmetry principles only, and they found that there appear 37 independent terms. These couplings cannot be determined by present experiments. Therefore, additional assumptions are needed to describe the weak interactions. There are two procedures available:

a) the "weak deformation model"

b) the "factorization model".

Both models can be formulated without any reference to resonances. Because the strong couplings of the $O(p^4)$ order in the chiral Lagrangian seem to be saturated by resonance exchange [8], it seems natural that the weak interactions at $O(p^4)$ order in the chiral Lagrangian might be explained by resonance contributions.

The purpose of this paper is to clarify the role of resonances in the counterterms for $K \to \pi e^+ e^-$ decays.

The paper is organized as follows: In Sect. 2 we shall repeat the general results for $K \to \pi\gamma^*$ and $K \to \pi\gamma\gamma$ amplitudes and we explain the electroweak counterterm-couplings by resonance exchange. In Sect. 3 we shall analyse these terms using "factorization model", "weak deformatoion models" and we shall apply the extended Nambu and Jona-Lasinio model for the couplings in the strong chiral Lagrangian of $O(p^4)$ order. In Sect. 4 the large N_c model is combined with the resonance saturation. The $K_L \to \pi^0 e^+ e^-$ decay rate is briefly analysed in Sect. 5. In Sect. 6 we give summary of our results.

2 $K \to \pi\gamma^*$ and $K \to \pi\gamma\gamma$ decays in CHPT

It was shown that in the chiral perturbation theory at $O(p^2)$ $K \to \pi\gamma^*$ transitions are forbidden for a virtual photon $\gamma^*(q)$ for any value of q^2 [1]. Combining the contributions coming from one-loop and the counterterms, induced by strong, weak and electromagnetic interactions, [2, 3], the amplitudes for $K^+ \to \pi^+\gamma^*$ and $K_S \to \pi^0\gamma^*$ can be written as

$$A(K^+ \to \pi^+\gamma^*) = \frac{G_8 e}{(4\pi)^2} q^2 \Phi_+(q^2) \epsilon^\mu (p' + p)_\mu \tag{1}$$

$$A(K_S^0 \to \pi^0\gamma^*) = \frac{G_8 e}{(4\pi)^2} q^2 \Phi_S(q^2) \epsilon^\mu (p' + p)_\mu \tag{2}$$

where p and p' are pion's and kaon's momenta, $G_8 = \sqrt{\frac{1}{2}} G_F s_1 c_1 c_3 g_8$ is defined in [1] and from $K \to \pi\pi$ it was found that $|g_8| = 5.1$. The calculated one-loop correction reduces this value to $|g_8^{loop}| = 4.3$ [5, 13]. The factors $\Phi_{+,S}$ are split into one-loop and counterterm contributions

$$\Phi_+ = W_+ + \Phi_K + \Phi_\pi \tag{3}$$

$$\Phi_S = W_S + 2\Phi_K \tag{4}$$

The loop contributions Φ_K and Φ_π are determined in [1, 2, 3]. The $W_{+,S}$ have been defined as [2, 3, 4]

$$W_+ = -\frac{16}{3}\pi^2 [W_1^r + 2W_2^r - 12L_9^r(\mu)] + \frac{1}{3} log \frac{\mu^2}{M_K M_\pi} \tag{5}$$

and

$$W_S = -\frac{16}{3}\pi^2[W_1^r - W_2^r] + \frac{1}{3}log\frac{\mu^2}{M_K^2} \tag{6}$$

In these equations $L_9^r(\mu)$ is a coupling constant at $O(p^4)$ strong Lagrangian defined in [8, 14, 15]

$$\begin{aligned} \mathcal{L}_4 &= ... - iTrL_9(f_+^{\alpha\beta}u_\alpha u_\beta) + \frac{1}{4}(L_{10} + 2H_1)Tr(f_{+\alpha\beta}f_+^{\alpha\beta}) \\ &- \frac{1}{4}(L_{10} - 2H_1)Tr(f_{-\alpha\beta}f_-^{\alpha\beta}) \end{aligned} \tag{7}$$

with $u^2 = U = exp(\frac{i}{f}\Sigma_i\lambda^i\phi_i)$ and $f \simeq f_\pi = 0.0933GeV$. The notation here is defined below:

$$f_\pm^{\alpha\beta} = uF_L^{\alpha\beta}u^\dagger \pm u^\dagger F_R^{\alpha\beta}u \tag{8}$$

$$F_L^{\alpha\beta} = \partial^\alpha l^\beta - \partial^\beta l^\alpha - i[l^\alpha, l^\beta] \tag{9}$$

$$F_R^{\alpha\beta} = \partial^\alpha r^\beta - \partial^\beta r^\alpha - i[r^\alpha, r^\beta] \tag{10}$$

$$u_\alpha = iu^\dagger D_\alpha U u^\dagger \tag{11}$$

$$D_\alpha = \partial_\alpha U - ir_\alpha U + iUl_\alpha \tag{12}$$

and l_α and r_α denote left and right matrix external fields, with spin 1. In the presence of the external electromagnetic fields only $l_\alpha = r_\alpha = |e|QA_\alpha$. The coupling constant L_9 is connected with the renormalized L_9^r as

$$L_9 = L_9^r + \Gamma_9\frac{\nu^{-\epsilon}}{(4\pi)^2\hat{\epsilon}}, \;\; \Gamma_9 = \frac{1}{4}. \tag{13}$$

In this paper we choose $\nu = m_\rho$ for the renormalization scale ν. In a similar way W_1 and W_2 counterterms of weak and electromagnetic origin can be written in the form [2]

$$W_1 \;=\; W_1^r + \Gamma_9 \frac{\nu^{-\epsilon}}{(4\pi)^2 \hat{\epsilon}}, \;\; \Gamma_9 = \frac{1}{4} \tag{14}$$

In ref. [2, 3] it was found that in the case of $K \to \pi \gamma^*$ and $K \to \pi\gamma\gamma$ decays there are three relevant local counterterms:

$$\begin{aligned} \mathcal{L}_4^{\Delta S=1,em} \;=\; & \frac{ieG_8 f^2}{2} F^{\mu\nu} [W_1 Tr(Q\lambda_{6-i7} U^\dagger D_\mu U U^\dagger D_\nu U) \\ + \;& W_4 Tr(QU^\dagger D_\mu U \lambda_{6-i7} U^\dagger D_\nu U)] \\ + \;& \frac{e^2 f^2}{2} G_8 F^{\mu\nu} F_{\mu\nu} W_4 Tr(\lambda_{6-i7} Q U^\dagger Q U) + h.c. \end{aligned} \tag{15}$$

The analysis of $K^+ \to \pi^+\gamma\gamma$ decay shows that there is the following combination of coupling constants

$$\hat{c} \;=\; 32\pi^2 [4(L_9 + L_{10}) - \frac{1}{3}(W_1 + 2W_2 + 2W_4)]. \tag{16}$$

The combination $L_9 + L_{10}$ and the weak contribution $W_1 + 2W_2 + 2W_4$ are a renormalization scale invariant [3, 14]. The loop contribution is finite in $K \to \pi\gamma\gamma$ decays and for $K^0 \to \pi^0\gamma\gamma$ decay $O(p^4)$ couterterms contribution vanishes.

In order to determine the resonance saturation of the counterterm couplings in the chiral Lagrangian at $O(p^4)$, the lowest order couplings in the chiral expanssion are needed. All these couplings are of $O(p^2)$, and resonance exchange will automatically produce contributions of $O(p^4)$. Using the equations of motion for resonances, the authors of ref. [8] have found that L_9, L_{10}

and H_1 get contributions from vector and axial-vector reconances. They are

$$\begin{aligned} L_9^V &= \frac{F_V G_V}{2M_V^2}, \;\; L_{10}^V = -\frac{F_V^2}{4M_V^2}, \\ H_1^V &= -\frac{F_V^2}{8M_V^2} \end{aligned} \tag{17}$$

$$L_9^A = 0, \;\; L_{10}^A = \frac{F_A^2}{4M_A^2}, \;\; H_1^V = -\frac{F_A^2}{8M_A^2} \tag{18}$$

where M_V and M_A are the octet masses of vector and axial-vector mesons. The octet couplings F_V and G_V can in principle be determined from the decay rates $\rho \to l^+ l^-$ and $\rho \to 2\pi$, respectively, and they are

$$|F_V| = 0.154 \;\; GeV, \;\; |G_V| = 0.069 \;\; GeV. \tag{19}$$

For the determination of F_A and the octet mass M_A in the chiral limit the authors of [8] have used Weinberg's sum rules [18]. These two sum rules connect axial vector meson with vector meson parameters,

$$F_V^2 = F_A^2 + f^2 \tag{20}$$

$$M_V^2 F_V^2 = M_A^2 F_A^2, \tag{21}$$

$$F_A^2 = 0.128 \;\; GeV \tag{22}$$

$$M_A^2 = 0.968 \;\; GeV \tag{23}$$

The relevant $O(p^4)$ couplings L_9, L_{10} and H_1 are in general divergent, like the rest of $O(p^4)$ couplings. They depend on the renormalization scale ν which is not seen in the observable quantities. The results we use, like [8, 14, 15] are obtained at $\nu = M_\rho$.

$$L_9^r = 6.9 \times 10^{-3} \tag{24}$$

$$L_{10}^r = -6.0 \times 10^{-3} \tag{25}$$

$$H_1^r = 7.0 \times 10^{-3} \tag{26}$$

The H_1^r is not accessible experimentally, but vector and axial vector mesons determine this coupling at scale $\nu = M_\rho$ and, once the form of strong chiral Lagrangian at $O(p^4)$ is chosen in the form [8, 14, 15], this term is fixed by resonance exchange.

3 Counterterms and models for the weak Lagrangian at $O(p^4)$ order

The weak deformation model and the factorization model are used in the literature [3, 9] towards constructing the weak Lagrangian of $O(p^4)$ order. Both models rest on the assumption that the strong chiral Lagrangian already determines the dominant features of $\Delta S = 1$ effective Lagrangian. It is obvious that such a procedure cannot be expected to yield the complete weak Lagrangian, since the short-distance contribution has no equivalence in the strong sector. One might expect that the dominant long-distance contribution results from resonances exchange like in the strong sector. We remark that there are two possible ways to obtain the resonance contribution to $\mathcal{L}_w$: One might first calculate the resonance contributions to the strong Lagrangian and then use a weak deformation or factorization procedure. Or, one might apply the weak deformation or factorization to the strong resonance Lagrangian of lowest order first and then integrate out the resonances, using their equations of motion. If the second approach is used, the resonance contribution recognized in H_1 counterterm of the Lagrangian (7), is undoubtfully present.

a) Weak deformation model

This model is inspired by the geometry of the coset space $G/SU(3)_V$ [4]. It can easily be obtained starting with the strong Lagrangian of $O(p^2)$

$$\mathcal{L}_2 = \frac{f^2}{4} Tr(u_\mu u^\mu), \tag{27}$$

and making the replacement

$$u_\mu \to u_\mu + G_8 f^2 \{u_\mu, \Delta\} - \frac{2}{3} G_8 f^2 Tr(u_\mu \Delta), \tag{28}$$

where

$$\Delta = u \lambda_6 u^\dagger. \tag{29}$$

It is useful to introduce two forms

$$l_\mu = u[\partial - i(v_\mu - a_\mu)]u^\dagger = \Gamma_\mu + \frac{1}{2} i u_\mu \tag{30}$$

$$r_\mu = u[\partial - i(v_\mu + a_\mu)]u^\dagger = \Gamma_\mu - \frac{1}{2} i u_\mu. \tag{31}$$

In this model Γ_μ is deformed to

$$\Gamma_\mu \to \Gamma_\mu + \frac{1}{2} i G_8 f_\pi^2 \{u_\mu, \Delta\} - \frac{1}{3} i G_8 f_\pi^2 Tr(u_\mu \Delta) \tag{32}$$

After performing the weak deformation on $\mathcal{L}_4$ in (7) the counterterm coupling defined in (15) are found to be

$$W_1^r = 4(L_9^r + L_{10}^r + 2H_1^r) \tag{33}$$

$$W_2^r = 4L_9^r \tag{34}$$

$$W_4^r = 4(L_{10}^r - H_1^r). \tag{35}$$

With the numerical results for L_9^r, L_{10}^r and H_1^r from (24), (25) and (26) we find

$$W_1^{WDM} = -0.0524 \tag{36}$$

$$W_2^{WDM} = 0.0276 \tag{37}$$

$$W_4^{WDM} = 0.004 \tag{38}$$

what leads, using (5) and (6), to $W_+^{WDM} = -5.01$ $W_S^{WDM} = 4.50$. Among three possible decays only the decay rate of $K^+ \to \pi^+ e^+ e^-$ has been observed. The experimental bound for the branching ratio $K^+ \to \pi^+ e^+ e^-$ is obtained from Brookhaven experiment [19]

$$BR(K^+ \to \pi^+ e^+ e^-) = (2.99 \pm 0.22) \times 10^{-7}, \tag{39}$$

resulting in bounds $W_+^{WDM} = 0.89^{+0.24}_{-0.14}$. It is obvious that the numerical value obtained by using the weak deformation model does not explain this experimental limit.

In the case of $K^+ \to \pi^+ \gamma\gamma$ the counterterm coupling defined in (16) is $\hat{c}^{WDM} = 0$. In ref. [5] the relation (34) was criticized as a result of assumption which is not part of the CHPT. We point out that this result originally was derived using the octet dominance hypothesis. Namely, the authors of ref.

[1] have noticed that the one-loop amplitudes for $K^+ \to \pi^+\gamma^*$, $K^0 \to \pi^0\gamma^*$ and $\eta \to \bar{K}^0\gamma^*$, if $m_K = m_\pi$, satisfy the relation

$$A(K^+ \to \pi^+\gamma^*)|_l \;=\; -\sqrt{2}A(K^0 \to \pi^0\gamma^*)|_l = \sqrt{\frac{2}{3}}A(\eta \to \bar{K}^0\gamma^*)|_l. \quad (40)$$

This relation restricts two pseudoscalars to be in the pure octet in the $SU(3)$ limit. We find that without $SU(3)$ symmetry these amplitudes satisfy

$$A(K^+ \to \pi^+\gamma^*)|_l \;=\; -\frac{1}{\sqrt{2}}(A(K^0 \to \pi^0\gamma^*)|_l + \sqrt{\frac{1}{3}}A(\eta \to \bar{K}^0\gamma^*)|_l). (41)$$

Generally, as it was noticed in ref. [1], it means that two pseudoscalars can be in state of representation 8, 10, and $\bar{1}0$ of $SU(3)$, and with the help of Clebsch-Gordon coefficients for 8×10 of $SU(3)$ group it can be seen that decuplet components cancel out in (41). Imposing (40), if $SU(3)$ limit holds, or (41) when $SU(3)$ is broken, on the amplitudes containing the weak counterterms, we find that relation (34) exists in both cases, with and without $SU(3)$ limit. Athough our result for W_+ disagrees with the experimental bound, we cannot rule out the weak deformation model. The model itself contains very important feature: the relations (33), (34) and (35) are scale independent [4] and further study of this model would be useful.

b) Factorization model

The effective $\Delta S = 1$ weak Hamiltonian is given by

$$H_{eff} \;=\; \frac{G}{\sqrt{2}}C_i(\mu^2)Q_i + h.c. \quad (42)$$

where Q_i are four-quark operators and $C_i(\mu^2)$ are Wilson coefficients depending on the QCD renormalization scale μ . To lowest order in CHPT the left-current is realized as

$$J_\mu^{(1)} = \frac{\delta S_2}{\delta l_\mu}, \tag{43}$$

where S_2 is the action determined by $\mathcal{L}_2$, what gives

$$\begin{aligned} J_\mu^{(1)} &= -\frac{f^2}{2} u^\dagger u_\mu u \\ &= -i\frac{f^2}{2}\{U^\dagger \partial_\mu U + ieA_\mu U^\dagger [U,Q]\}. \end{aligned} \tag{44}$$

To $O(p^4)$, the factorization model is defined for the dominant octet part of the weak Lagrangian by

$$\mathcal{L}_{W4}^{FM} = 4g_8 Tr(\lambda_{6-i7}\{J_\mu^{(1)}, J^{\mu(3)}\}), \tag{45}$$

where $J_\mu^{(3)}$ is determined from

$$\mathcal{J}_\mu^{(3)} = \frac{\delta S_4}{\delta l_\mu}. \tag{46}$$

In our case it leads to

$$\begin{aligned} \mathcal{J}_\mu^{(3)} &= -L_9\{eF_{\mu\nu}([L^\nu, U^\dagger Q U] + [L^\nu, Q])\} \\ &+ iL_9 \partial^\nu \{[L_\mu, L_\nu] + ieA_\mu(-[L_\nu, Q] + [L_\nu, U^\dagger Q U])\} \\ &+ 2L_{10}\partial^\nu(eF_{\mu\nu}U^\dagger Q U) + 4H_1 \partial^\nu(eF_{\mu\nu}Q) \end{aligned} \tag{47}$$

where $L_\mu = U^\dagger \partial_\mu U$. Using these two currents, we easily find

$$W_1^r = 8(L_9^r + L_{10}^r + 2H_1^r) \tag{48}$$

$$W_2^r = 8L_9^r \tag{49}$$

$$W_4^r = 8(L_{10}^r - H_1^r) \tag{50}$$

With the help of the numerical results for L_9^r, L_{10}^r and H_1^r from (24), (25) and (26) we derive

$$W_1^{FM} = -0.1048 \tag{51}$$

$$W_2^{FM} = 0.0552 \tag{52}$$

$$W_4^{FM} = 0.008, \tag{53}$$

what results in $W_+^{FM} = -4.87$ and $W_S^{FM} = 8.67$. The result for W_+, like the one calculated in the case of the weak deformation model, disagees with the experimental one. The combination of the counterterm couplings derived in (16) gives $\hat{c}^{FM} = -1.14$. At present there is only an upper bound for the branching ratio of this process $BR(K^+ \to \pi^+\gamma\gamma) < 1 \cdot 10^{-6}$ [21] . This upper bound and the analysis of ref. [3] lead to the limits $-7.02 < \hat{c} < 1.89$ and the obtained value for $\hat{c}^{FM} = -1.14$ is allowed by this bounds.

Now, we search for the model which might accommodate this experimental result and we apply the extended Nambu and Jona-Lasinio model described in ref. [20]. In this reference, the low-energy effective action of an extended Nambu and Jona-Lasinio model to $O(p^4)$ in the chiral counting is derived. The couplings of our interest are L_9, L_{10} and H_1. It was found [20] that these couplings can be expressed as

$$L_9 = \frac{N_c}{16\pi^2}\frac{1}{6}[(1-g_A^2)\Gamma_0(1+\gamma_{03}) + 2g_A^2\Gamma_1(1+\frac{3}{2}\gamma_{12} - \frac{1}{2}\gamma_{13})] \tag{54}$$

$$L_{10} = -\frac{N_c}{16\pi^2}\frac{1}{6}[(1-g_A^2)\Gamma_0(1+\gamma_{03}) + g_A^2\Gamma_1(1+\gamma_{13})] \tag{55}$$

$$H_1 = -\frac{N_c}{16\pi^2}\frac{1}{12}[(1-g_A^2)\Gamma_0(1+\gamma_{03}) - g_A^2\Gamma_1(1+\gamma_{13})]. \tag{56}$$

In these equations we use

$$\gamma_{03} = -\frac{\Gamma(2,x)}{\Gamma(0,x)}\frac{3}{5}g \tag{57}$$

$$\gamma_{12} = \frac{\Gamma(3,x)}{\Gamma(1,x)}\frac{1}{5}g \tag{58}$$

$$\gamma_{13} = -\frac{\Gamma(3,x)}{\Gamma(1,x)}\frac{3}{5}g, \tag{59}$$

where $\Gamma(n,x)$ denotes the incomplete gamma function

$$\Gamma(n-2, x = \frac{M_Q^2}{\lambda_\chi}) = \int_{\frac{M_Q^2}{\lambda_\chi^2}}^{\infty} \frac{dz}{z} e^{-z} z^{n-2}, \;\; n = 1,2,3,..., \tag{60}$$

g_A can be identified as a constant of the constituent chiral quark model and g might be connected to the gluon vacuum condensate [20], λ_χ is a cut-off scale, M_Q is the constituent chiral quark-mass. For the most favorable set of parameters $g_A = 0.61$, $M_Q = 0.265 GeV$ and $\lambda_\chi = 1.165 GeV$ [20], it is easy to calculate $L_9^r = 7.0 \cdot 10^{-3}$, $L_{10}^r = -5.9 \cdot 10^{-3}$ and $H_1^r = -4.7 \cdot 10^{-3}$. These values lead to

$$W_1^{WDM} = -0.0327 \tag{61}$$

$$W_2^{WDM} = 0.028 \tag{62}$$

$$W_4^{WDM} = -0.0049 \tag{63}$$

and consequently $W_+^{WDM} = 3.91$, $W_S^{WDM} = 6.69$ and $\hat{c}^{WDM} = 0$. For the factorization model the values of W_i are doubled and physically relevant values are $W_+^{FM} = 2.69$, $W_S^{FM} = 6.69$ and $\hat{c}^{FM} = -1.41$. The numerical values of W_i are calculated at the scale $\nu = m_\rho$. In Table 1 we present W_+, W_S and $\hat{c}$ counterterms calculated for some of the values of the scale ν. Analysing fits from ref. [20] we find that their fit. 4 gives most favorable value of the W_+. In this fit there is no explicit vector (axial-vector) degree of freedom. Using this fit, it was found $L_9^r = 5.8 \cdot 10^{-3}$, $L_{10}^r = -5.1 \cdot 10^{-3}$ and $H_1^r = -2.4 \cdot 10^{-3}$. These values leads to

$$W_1^{FM} = -0.0328 \tag{64}$$

$$W_2^{FM} = 0.0464 \tag{65}$$

$$W_4^{FM} = -0.0216, \tag{66}$$

resulting in $W_+^{FM} = 1.22$, $W_S^{FM} = 4.46$ and $\hat{c}^{FM} = -0.88$. for the scale $\nu = m_\rho$. For the scale $\nu = 0.265$ GeV these values are $W_+^{FM} = 0.51$, $W_S^{FM} = 3.75$ and $\hat{c}^{FM} = -0.88$. The strenght of the counterterm coupling $\hat{c}^{FM} = -0.88$ is within the experimental limits. We can conclude that the factorization model combined with the parameters used in the extended Nambu and Jona-Lasinio model can accommodate the experimentally measured decay rate $BR(K^+ \to \pi^+ e^+ e^-)$ and the bounds on conterterm coupling $\hat{c}$ obtained from $BR(K^+ \to \pi^+ \gamma\gamma)$ decay rate.

4 Large N_c limit and counterterms

Using the large N_c limit, $\Delta S = 1$ effective Hamiltonian is given by [16, 17]

$$H_{eff}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* Q_2 + h.c. \tag{67}$$

where $Q_2 = 4(\bar{s}_L \gamma_\mu u_L)(\bar{u}_L \gamma_\mu d_L)$. The full effective Hamiltonian at the leading order of large N_c is

$$H_{eff}^{\Delta S=1} = 4(G_8^{(\frac{1}{2})} H_8^{(\frac{1}{2})} + G_{27}^{(\frac{1}{2})} H_{27}^{(\frac{1}{2})} + G_{27}^{(\frac{3}{2})} H_{27}^{(\frac{3}{2})}), \tag{68}$$

where in the strict large-N_c $g_8^{(\frac{1}{2})}|_{N_c\to\infty} = \frac{3}{5}$, $g_{27}^{(\frac{1}{2})}|_{N_c\to\infty} = \frac{1}{15}$, $g_{27}^{(\frac{3}{2})}|_{N_c\to\infty} = \frac{1}{3}$. In this expression

$$H_{27} = -\frac{2}{3}(L_\mu)_{21}(L^\mu)_{13} - (L_\mu)_{23}(L^\mu)_{11}. \tag{69}$$

H_{27} induces both $|\Delta I| = \frac{1}{2}$ and $|\Delta I| = \frac{3}{2}$ transitions via its components

$$H_{27} = \frac{1}{9} H_{27}^{(\frac{1}{2})} + \frac{5}{9} H_{27}^{(\frac{3}{2})}. \tag{70}$$

In our calculation we kept both components of Q_2 with $|\Delta I| = \frac{1}{2}$ and $|\Delta I| = \frac{3}{2}$, the octet and twenty-sevenplet . With the use of the expressions (43) and (47) we derive

$$A(K^+ \to \pi^+ \gamma^*) = \sqrt{\frac{1}{2}} G_F s_1 c_1 c_3 4 L_9 q^2 \epsilon^\mu (p' + p)_\mu \tag{71}$$

$$A(K^0 \to \pi^0 \gamma^*) = 0. \tag{72}$$

For the $K^+ \to \pi^+ \gamma\gamma$ decay we obtain

$$G_8 \hat{c} = -32\pi^2 \sqrt{\frac{1}{2}} G_F s_1 c_1 c_3 4(L_9 + L_{10}). \tag{73}$$

The authors of ref. [16] have calculated the octet component of the isospin $\frac{1}{2}$ effective Hamiltonian. In the large N_c limit the G_8 coupling is $g_8 = \frac{3}{5}$ (to be compared with the experimental value 5.1) They found that the factorization of the $G_8 W_i$, where $i = 1, 2, 4$, calculated at order $O(p^4)$ of the chiral Lagrangian is not valid at order N_c in the $\frac{1}{N_c}$- expansion. We confirm their result

$$W_1 = W_2 \tag{74}$$

$$G_8 W_2 = 8G_8|_{\frac{1}{N_c}} L_9 \tag{75}$$

$$G_8 W_4 = 12G_8|_{\frac{1}{N_c}} L_{10}. \tag{76}$$

where the subscript $\frac{1}{N_c}$ explains that the given coupling constant is determined at the leading order of the $\frac{1}{N_c}$ expansion. We derive numerical results, taking into account the values of L_9 and L_{10} when they are dominated by resonance exchange (24) and (25). They correspond to the expressions of $G_8 W_i$

$$G_8 W_1 \rightarrow \sqrt{\frac{1}{2}} G_F s_1 c_1 c_3 0.0552 \tag{77}$$

$$G_8 W_2 \rightarrow \sqrt{\frac{1}{2}} G_F s_1 c_1 c_3 0.0552 \tag{78}$$

$$G_8 W_4 \rightarrow -\sqrt{\frac{1}{2}} G_F s_1 c_1 c_3 0.072 \tag{79}$$

or $G_8 W_S \rightarrow -\sqrt{\frac{1}{2}} G_F s_1 c_1 c_3 0.237$, and $G_8 W_+ \rightarrow -\sqrt{\frac{1}{2}} G_F s_1 c_1 c_3 0.737$. The result corresponding to $G_8 W_+$ is quite close to the lower experimental bound.

The result which corresponds to $\sqrt{\frac{1}{2}}G_F s_1 c_1 c_3 \hat{c}$ is also within the experimental bound $G_8 \hat{c} = -\sqrt{\frac{1}{2}}G_F s_1 c_1 c_3 1.01$. The analysis of ref. [16] contains the effective the Hamiltonian calculated at the order p^4 in the chiral expansion and at the order N_c in the $\frac{1}{N_c}$-expansion. Their result leads to the conclusion that the factorization in $G_8 W_i$ is not valid when next-to-leading corrections in $\frac{1}{N_c}$ are calculated. Here, we have modified the leading term in $\frac{1}{N_c}$-expansion taking into account the contribution of 27 component of the Q_2 operator. Results are in slightly better agreement (factor of $\frac{5}{3}$ increase) than the ones derived in [16] for their leading term in $\frac{1}{N_c}$-expansion. It is interesting in this approach the counterterm contributions do not depend on H_1 when calculated at the leading order in $\frac{1}{N_c}$.

5 $K_L \to \pi^0 e^+ e^-$ and CP violation

The decay $K_L \to \pi^0 e^+ e^-$ is being investigated as a signal of direct $\Delta S = 1$ CP violation. In addition to a CP conserving process, which proceeds through two photon exchanges, there are two kinds of the CP violating decay: one proportional to the well known parameter ϵ and the other direct CP violating effect. The direct CP violating component is the simplest. Uncertainties are coming from poor knowledge of the Standard model parameters, top quark mass and CKM matrix elements. The QCD short distance corrections to this mode have been analyzed to next-to-leading order by Buras et al. [7]. The weak operator responsible for this decay is

$$\mathcal{H}_w = \frac{G_F}{\sqrt{2}}[C_{7V} + C_{7A}], \tag{80}$$

where

$$Q_{7V} = (\bar{s}d)_{V-A}(\bar{e}e)_V \tag{81}$$

$$Q_{7A} = (\bar{s}d)_{V-A}(\bar{e}e)_A. \tag{82}$$

The amplitude for $K_L \to \pi^0 e^+ e^-$ can be written as [2, 5]

$$A(K_L \to \pi^0 e^+ e^-) = -\frac{G_8 \alpha}{4\pi}(p+p')^\mu \bar{u}[\phi_V \gamma_\mu + \phi_A \gamma_\mu \gamma_5]v \tag{83}$$

with

$$\phi_V = \epsilon \phi_S - \frac{16\pi^2}{3} i Im W_1 \tag{84}$$

and

$$\phi_A = \frac{16\pi^2}{3} i Im W_5. \tag{85}$$

W_5 is defined by the effective Lagrangian counterterm

$$\mathcal{L}'_w = -\frac{i2\pi\alpha}{3} G_8 W_5 \bar{e}\gamma^\mu \gamma_5 e Tr(\lambda_{6-i7})(\partial U - ie[A_\mu, U])U^\dagger. \tag{86}$$

Taking the ϕ_V and ϕ_A as in [5, 7]

$$\phi_V = e^{i\pi/4}(0.57 \times 10^{-3}) - i1.0 \times 10^{-3} \tag{87}$$

and

$$\phi_A = i1.0 \times 10^{-3}, \tag{88}$$

we calculate the branching ratio $BR(K_L \to \pi^0 e^+ e^-)$ for the counterterms couplings. For $W_+ = 1.22$ and $W_S = 4.46$, obtained in the case of the extended Nambu and Jona - Lasinio model, we find $BR(K_L \to \pi^0 e^+ e^-) =$

$1.15 \cdot 10^{-10}$ and for $W_+ = 0.51$ and $W_S = 3.75$ the decay rate is $BR(K_L \rightarrow \pi^0 e^+ e^-) = 7.77 \cdot 10^{-11}$. From the large N_c approach to the CP violating process we get $BR(K_L \rightarrow \pi^0 e^+ e^-) = 4.79 \cdot 10^{-9}$. Experimentally was determined only the upper limit $BR(K_L \rightarrow \pi^0 e^+ e^-) \leq 4.3 \cdot 10^{-9}$, a result from [22], and $BR(K_L \rightarrow \pi^0 e^+ e^-) \leq 4.3 \cdot 10^{-11}$ obtained by [23]. From our analysis it follows that indirect CP violation is rather dependent on the W_S counterterm coupling. It was noticed [5] that it is very difficult to distinguish between direct and indirect CP violation. These authors point out that the existence of direct CP violating term in the branching ratio $BR(K_L \rightarrow \pi^0 e^+ e^-)$ can not be seen without observing branching ratio $BR(K_S \rightarrow \pi^0 e^+ e^-)$. The CP conserving process proceeds via two photon exchange. The contribution of vector and scalar mesons to $K_L \rightarrow \pi^0 \gamma\gamma$ is completely determined to $O(p^6)$ [4, 24]. The authors of ref. [5] made a fit to the $\gamma\gamma$ spectrum in $K_L \rightarrow \pi^0 \gamma\gamma$ in order to fix the parameter describing the role of vector mesons at this order. They obtain rather large value of the relevant parameter a_V, comparing the results when the weak deformation model was used like in [4, 24].

6 Summary

We have investigated counterterm couplings required by CHPT in the decays $K^+ \rightarrow \pi^+ e^+ e^-$, $K_S \rightarrow \pi^0 e^+ e^-$ and $K_L \rightarrow \pi^0 e^+ e^-$. The counterterms induced by the strong, weak and electromagnetic interactions have been fixed by resonance exchange.

- In our approach the weak deformation model, used for the weak interac-

tions, cannot reproduce the experimental result for $K^+ \to \pi^+ e^+ e^-$ decay rate.

- The branching ratio $BR(K^+ \to \pi^+ e^+ e^-)$ and $\hat{c}$ defined in $(K^+ \to \pi^+ \gamma\gamma)$ decay amplitude, are in agreement with the experimental results, when the factorization model combined with the set of the parameters determined in the extended Nambu and Jona-Lasinio model is applied.

- The counterterm couplings depend on a physically unobservable coupling H_1. At the leading order in large N_c there is no dependence on the H_1 coupling.

- The $BR(K^L \to \pi^0 e^+ e^-)$ can be predicted within these approaches. The decay rate is very parameter dependent.

Acknowledgements I would like to thank E. de Rafael for a motivation of this work and for many useful and stimulating discussions. I also thank CNRS for the financial support during my visit to University of Marseille.

ν	m_ρ	$0.265 GeV$	$1.165 GeV$	m_ρ	$0.265 GeV$	$1.165 GeV$
W_+	3.91	3.20	4.19	2.69	1.96	4.19
W_S	3.49	2.78	3.77	6.69	5.98	6.96
c	0	0	0	-1.41	-1.41	-1.41

Table 1: The counterterms W_+, W_S calculated using the extended Nambu and Jona-Lasinio model for these values of the renormalization scale. The first three columns contain the values in the case of weak deformation model and in the last three columns there are values obtained in the case of factorization model.

References

[1] G. Ecker, A. Pich and E. de Rafael, Phys. Lett. **B 189** (1987) 363.

[2] G. Ecker, A. Pich and E. de Rafael, Nucl. Phys. **B 291** (1987) 692.

[3] G. Ecker, A. Pich and E. de Rafael, Nucl. Phys. **B 303** (1988) 665.

[4] G. Ecker, A. Pich and E. de Rafael, Phys. Lett. **B 237** (1990) 481.

[5] John F. Donoghue and F. Gabbiani, Phys. Rev **D 51** (1995) 2187.

[6] G. D' Ambrossio, G. Ecker, G. Isidori and H. Neufeld, preprint IFNNA-IV-94/24, DSF-94/24 UWThPh-1994-20, Oct. 1994.

[7] A. J. Buras, M. E. Lautenbacher, M. Misiak and M. Münz, Nucl. Phys. **B423** (1994) 349.

[8] G. Ecker, J. Gasser, A. Pich and E. de Rafael, Nucl. Phys. **B 321** (1989) 311.

[9] G. Ecker, J. Kambor and D. Wyler, Nucl. Phys. **B 394** (1993) 101.

[10] G. Isidori and A. Pugliese, Nucl. Phys. **B385** (1993) 437.

[11] S. Fajfer, Z. Phys **C 61** (1994) 645.

[12] J. Kambor, J. Missimer and D. Wyler, Nucl. Phys. **B 346** (1990) 17.

[13] J. Kambor, J. Missimer and D. Wyler, Phys. Lett. **B 261** (1991) 496.

[14] J. Gasser and H. Leutwyler, Nucl. Phys. **B 321** (1985) 465, 517.

[15] J. Bijnens and F. Cornet, Nucl. Phys. **B296** (1986) 557.

[16] C. Bruno and J. Prades, Z. Phys. **C 57** (1993) 585.

[17] A. Pich, E. de Rafael, Nucl. Phys. **B358** (1991) 341.

[18] S. Weinberg, Phys. Rev. Lett. **18** (1967) 507.

[19] C. Alliegro et al., Phys. Rev. Lett. **68** (1992) 278.

[20] J. Bijnens, C. Bruno and E. de Rafael, Nucl. Phys. **B390** (1993) 501.

[21] M. S. Atiya et al., (BNL-E787), Phys. Lett. **65** (1990) 1188.

[22] D. A. Harrris et al., Phys. Rev. Lett. **71** (1993) 3918.

[23] K. E. Ohl et al., Phys. Rev. Lett. **64** (1990) 2755.

[24] S. Fajfer, Phys. Rev. **D 51** (1995) 1101.